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ON THE FUNDAMENTAL PROPERTY OF THE LINEAR GROUP OF TRANSFORMATION IN THE PLANE.

By Dr. Arnold Emch, Lawrence, Kas.

A general projective transformation in the plane can easily be executed by means of two conics K and K' tangent to a certain straight line l in the following manner* (Fig. 1).

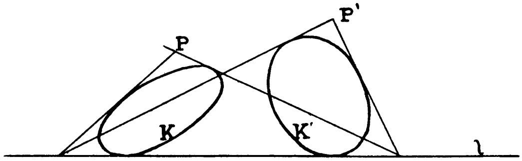

Figure 1.

From the point P to be transformed draw the two tangents to the conic K, and from their points of intersection with the line l draw the two possible tangents to the conic K'. The point of intersection of these two tangents is the point P' corresponding to P in the transformation. The invariant triangle of the transformation is obtained by the three other common tangents of the conics K and K'.

Now it is known that the linear transformation leaves the line at infinity invariant. Consequently, in a linear transformation, the conics K and K' must be parabolas.

By construction, or from the fact that every point of the line at infinity is transformed into another point of the line at infinity it follows that parallel lines are transformed into parallel lines. This property of the linear transformation is sufficient to prove in a simple way the well known theorem :

If in a projective transformation of the plane parallel lines are transformed into parallel lines, the areas of any two corresponding closed figures have a constant ratio.

To prove this we can consider two corresponding triangles Δ and Δ', however small (ABC and $A'B'C'$ in Fig. 2). Through each vertex of these triangles draw a line parallel to the opposite side and complete the net formed by parallels as is indicated in the figure.

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* For this proposition we refer to an unpublished paper of Prof. Newson.

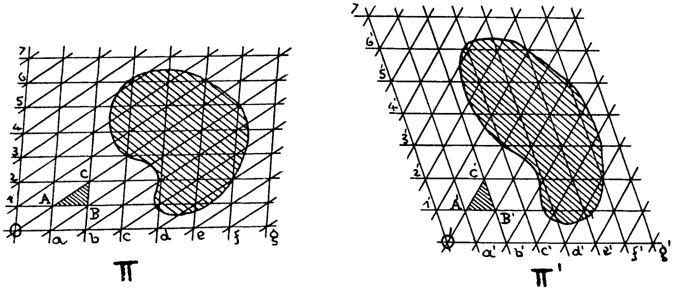

FIGURE 2.

Evidently, the points

$$O \text{ and } O'$$

$$a,\ b,\ c,\ d,\ e,\ f,\ g,\ \ldots$$

and

$$a',\ b',\ c',\ d',\ e',\ f',\ g',\ \ldots;$$

$$1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ \ldots$$

and

$$1',\ 2',\ 3',\ 4',\ 5',\ 6',\ 7',\ \ldots;$$

and the systems of parallel lines through these points are corresponding points and systems in the transformation.

Thus, the plane Π is divided into a net of equal triangles and the corresponding plane Π' into a net of corresponding equal triangles, such that any two corresponding triangles, or closed figures consisting of corresponding triangles, have the same constant ratio.

From this follows that any closed curve in the plane Π includes the same number of primitive triangles and parts of such triangles as the corresponding curve in the plane Π'. Designating the number of integral triangles within the curves by n, the sum of the fractional triangles within the curves by R and R', respectively, and the constant ratio by k, there is

$$\frac{n\triangle + R}{n\triangle' + R'} = k\,, \quad \text{or} \quad \frac{\triangle + \dfrac{R}{n}}{\triangle' + \dfrac{R'}{n}} = k\,.^*$$

Taking the limits, i. e., n infinitely large, or the triangles Δ and Δ' smaller than any finite quantity, this ratio becomes

$$\frac{\triangle}{\triangle'} = k\,, \text{ which was to be shown.}$$

* Compare Dirichlet's *Vorlesungen über Zahlentheorie*, 4th Ed., § 120, p. 310.